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Equations of Motion for Dynamically Stable Mobile Manipulators

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1 Introduction

This paper derives the equations of motion for Sparky, a mobile manipulator robot show in Fig. 1. These equations are used in the manipulation analysis, simulation, and experiments of [1].

2 Newton-Euler Equations

Assumptions

- Coulomb Friction
- Nonzero velocity in x
- Constant acceleration a
- Zero angular acceleration and angular velocity of link and object
- Wheels are very light and have a negligible moment of inertia

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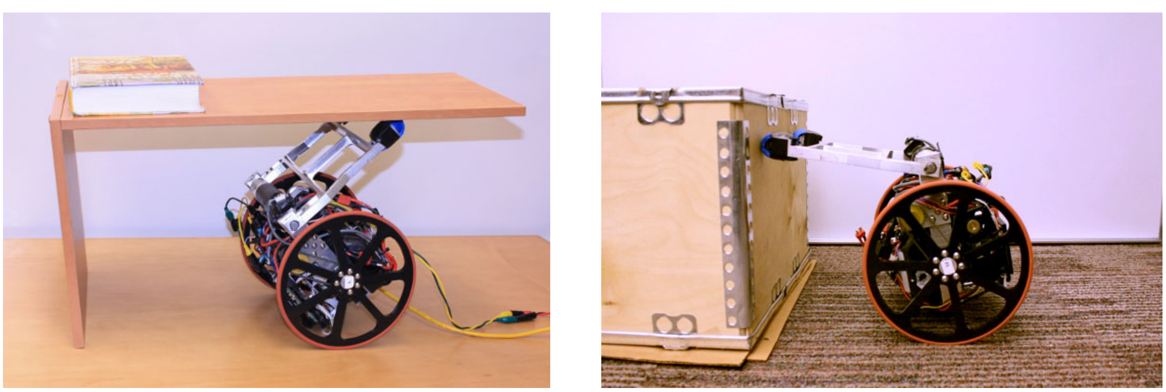


(a) (b)

Figure 1: Mobile Manipulator Robot Sparky

Symbol	Meaning
N, L, G, O	point
θ, ϕ, γ	angle
$r_\star$, $e_\star$, $l_\star$, ℓ	length
m_i	mass of body i
M	mass of box
a	linear acceleration
$F_\star$, $N_\star$	force
α	angular acceleration
M_A	moment about point A
T	Torque
$\mu_\star$	coefficient of friction

Table 1: Summary of Symbols

Wheel Free Body Diagram in Fig. 2(a).

$$\sum F_x = 2m_1 a = N_x + F_{Wx} \tag{1}$$
$$\sum F_y = 0 = -N_y - F_{g1} + F_{Wy} \tag{2}$$
$$\sum M_N = 2J_W \alpha_w = 0 = -T + r_1 F_{Wx} \tag{3}$$

where m_1 is the mass of each wheel (there are two).

Link Free Body Diagram in Fig. 2(b).

$$\sum F_x = m_2 a = -N_x - F_{Lx} \tag{4}$$
$$\sum F_y = 0 = N_y - F_{g2} + F_{y2} - F_{Ly} \tag{5}$$
$$\sum M_N = 0 = T - r_2(\sin\phi F_{g2} + \cos\phi m_2 a) + l_2(\cos\theta F_{Lx} - \sin\theta F_{Ly}) \tag{6}$$

Object Free Body Diagram in Fig. 2(c) and Fig. 2(d).

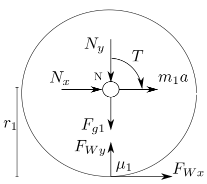
$$\sum F_x = Ma = F_{Lx} + F_{Ox} \tag{7}$$
$$\sum F_y = 0 = F_{Ly} - F_G + F_{Oy} \tag{8}$$
$$\sum M_G = 0 = -F_{Lx} l_L - F_{Ly}\frac{e_x}{2} + F_{Oy}\frac{e_x}{2} - F_{Ox}\frac{e_y}{2} \tag{9}$$

Where l_L is the y distance from L to G.

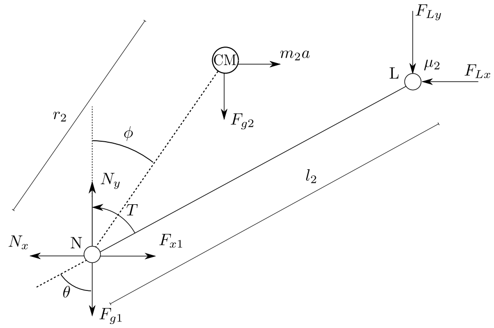
3 Equation Analysis

Expression for F_{Lx}

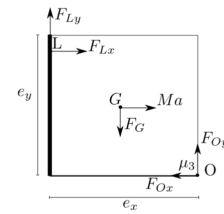
1. Start with Eq. 7.
$$Ma = F_{Lx} - F_{Ox}$$
2. Replace F_{Ox} with Coulomb friction.
$$Ma = F_{Lx} - \mu_3 F_{Oy}$$



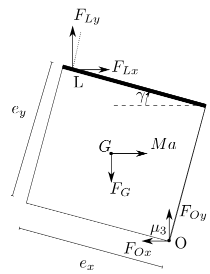
(a) Robot Wheel



(b) Robot Torso-link



(c) Object



(d) Object

Figure 2: Free-Body Diagram. (b) represents the robot by a single torso link. (c) refers to an object being pushed using SP. (d) presents the object for LP and LL. Thick lines indicate the surfaces of the object that are in contact with the robot.

3. Replace F_{Oy} using Eq. (8).

$$Ma = F_{Lx} - \mu_3(F_G - F_{Ly})$$

4. Reorder

$$a = \frac{F_{Lx} - \mu_3(F_G - F_{Ly})}{M} \tag{10}$$

This Eq. (10) *is Eq.* (1) *in [1].*

Initial Expression for a

1. Start with Eq. (4) and reorder terms.

$$F_{Lx} = -m_2a - N_x$$

2. Replace N_x using Eq. (1).

$$F_{Lx} = -m_2a - 2m_1a + F_{Wx}$$

3. Replace F_{Wx} using Eq. (3) and factor.

$$F_{Lx} = -a(2m_1 + m_2) + \frac{T}{r_1} \tag{11}$$

This Eq. (11) *is Eq.* (2) *in [1].*

Expression for F_{Ly} Take Eq. (6) and reorder.

$$F_{Ly} = \frac{T - r_2(\sin\phi F_{g2} + \cos\phi m_2 a) + l_2\cos\theta F_{Lx}}{l_2 \sin\theta} \tag{12}$$

This Eq. (12) *is Eq.* (3) *in [1].*

Final Expression for a

1. Take Eq. (10) and replace F_{Ly} with Eq. (12).

$$a = \frac{F_{Lx} - \mu_3\left(F_G - \frac{T - r_2(\sin\phi F_{g2} + \cos\phi m_2 a) + l_2\cos\theta F_{Lx}}{l_2\sin\theta}\right)}{M}$$

2. Distribute M.

$$a = \frac{F_{Lx} - \mu_3 F_G}{M} + \mu_3 \frac{T - r_2(\sin\phi F_{g2} + \cos\phi m_2 a) + l_2\cos\theta F_{Lx}}{M l_2 \sin\theta}$$

3. Replace F_{Lx} with Eq. (11).

$$a = \frac{-a(2m_1 + m_2) + \frac{T}{r_1} - \mu_3 F_G}{M} + \mu_3 \frac{T - r_2(\sin\phi F_{g2} + \cos\phi m_2 a) + l_2\cos\theta(-a(2m_1 + m_2) + \frac{T}{r_1})}{M l_2 \sin\theta}$$

4. Factor a.

$$a = \frac{-a(2m_1 + m_2)}{M} + \frac{\frac{T}{r_1} - \mu_3 F_G}{M} - \frac{a\mu_3 l_2\cos\theta(2m_1 + m_3)}{M l_2\sin\theta} + \mu_3 \frac{T - r_2(\sin\phi F_{g_2} + \cos\phi_2 a) + l_2\cos\theta \frac{T_1}{r}}{M l_2\sin\theta}$$

5. Solve for a.

$$a = \frac{\frac{\frac{T}{r_1} - \mu_3 F_G}{M} + \mu_3 \frac{T - r_2(\sin\phi F_{g_2} + \cos\phi_2 a) + l_2\cos\theta\frac{T_1}{r}}{M l_2\sin\theta}}{1 + \frac{2m_1 + m_2}{M} + \frac{\mu_3 l_2\cos\theta(2m_1 + m_2)}{M l_2\sin\theta}} \tag{13}$$

Eq.(13) gives an expression for acceleration based on the control input parameters θ, ϕ, T.

4 Push-Pull Comparison

Assumptions

- Square Box
- Point L is at top corner of box opposite to O

Derivation

1. Take Eq. (9) for square box with corner contact

$$0 = F_{oy}\ell - F_{ox}\ell - F_{Ly}\ell - F_{Lx}\ell$$

2. Divide through by ℓ and shift F_{Ly}

$$F_{Ly} = F_{oy} - F_{ox} - F_{Lx}$$

3. Pushing Case

(a) Replace F_{Ox} with Coulomb friction

$$F_{Ly} = F_{oy}(1-\mu_3) - F_{Lx}$$

(b) Consider an infinitesimal increase in pushing F_{Lx}, dF, whose sign is negative because it exerts a negative moment.

$$F_{Ly} = F_{oy}(1-\mu_3) - F_{Lx} - dF \tag{14}$$

4. Pulling Case

(a) Replace F_{Ox} with Coulomb friction

$$F_{Ly} = F_{oy}(1+\mu_3) - F_{Lx} \tag{15}$$

This Eq. (15) *is Eq.* (12) *in [1].*

(b) Consider an infinitesimal increase in pushing F_{Lx}, dF, whose sign is positive because it exerts a negative moment.

$$F_{Ly} = F_{oy}(1+\mu_3) - F_{Lx} + dF \tag{16}$$

References

[1] P. Kolhe, N. Dantam, and M. Stilman. Dynamic Pushing Strategies for Dynamically Stabile Mobile Manipulators. In *2010 IEEE International Conference on Robotics and Automation, 2010. Proceedings*, 2010.