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Worksheet: L11 – Rotations

CSCI-534: Robot Planning & Manipulation

Spring 2020

http://www.neil.dantam.name/rpm/B11-rotation.pdf

1. **Euler's Formula:** Prove that $e^{\pi\sqrt{-1}} = -1$

2. **Multiplication of Quaternion Units:** Simplify multiplication of pairs of quaternion units:

   (a) $\hat{\boldsymbol{\imath}}\hat{\boldsymbol{\jmath}}$

   $$\hat{\boldsymbol{\imath}}\hat{\boldsymbol{\jmath}}\hat{\boldsymbol{k}} = -1 \quad \rightsquigarrow \quad \hat{\boldsymbol{\imath}}\hat{\boldsymbol{\jmath}}\hat{\boldsymbol{k}}^2 = -\hat{\boldsymbol{k}} \quad \rightsquigarrow \quad -\hat{\boldsymbol{\imath}}\hat{\boldsymbol{\jmath}} = -\hat{\boldsymbol{k}} \quad \rightsquigarrow \quad \hat{\boldsymbol{\imath}}\hat{\boldsymbol{\jmath}} = \hat{\boldsymbol{k}}$$

   (b) $\hat{\boldsymbol{\jmath}}\hat{\boldsymbol{k}}$

   (c) $\hat{\boldsymbol{\imath}}\hat{\boldsymbol{k}}$

   (d) $\hat{\boldsymbol{\jmath}}\hat{\boldsymbol{\imath}}$

   (e) $\hat{\boldsymbol{k}}\hat{\boldsymbol{\jmath}}$

   (f) $\hat{\boldsymbol{k}}\hat{\boldsymbol{\imath}}$

3. **Quaternion Multiplication:** Simplify $(a_w + a_x\hat{\boldsymbol{\imath}} + a_y\hat{\boldsymbol{\jmath}} + a_z\hat{\boldsymbol{k}}) \otimes (b_w + b_x\hat{\boldsymbol{\imath}} + b_y\hat{\boldsymbol{\jmath}} + b_z\hat{\boldsymbol{k}})$

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4. **Quaternion Multiplication Commutativity:** Is quaternion multiplication commutative? Prove yes or no.

5. **Quaternion Rotations:** Convert the following axis-angle forms to unit quaternions:

   (a) $\theta = \pi$ and $\hat{\boldsymbol{u}} = \hat{\boldsymbol{\imath}}$
   $\exp\left(\frac{\pi}{2}\hat{\boldsymbol{\imath}}\right) \quad \rightsquigarrow \quad \sin\frac{\pi}{2}\hat{\boldsymbol{\imath}} + \cos\frac{\pi}{2} \quad \rightsquigarrow \quad \hat{\boldsymbol{\imath}}$

   (b) $\theta = \frac{\pi}{2}$ and $\hat{\boldsymbol{u}} = \hat{\boldsymbol{k}}$

   (c) $\theta = -\frac{3\pi}{2}$ and $\hat{\boldsymbol{u}} = \hat{\boldsymbol{k}}$

   (d) $\theta = 0$ and $\hat{\boldsymbol{u}} = \hat{\boldsymbol{\imath}}$